GAGE

Strength of
Corrugated Iron

Civil Engineering
B. S.

1903

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STRENGTH OF CORRUGATED IRON.

By
Ralph Hawes Gage.

Thesis for Degree of Bachelor of Science
in Civil Engineering.

College of Engineering.
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Presented June 1903.

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

RALPH HAMES GAGE

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IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE

OF Bachelor of Science in Civil Engineering.

Ira O. Baker.

HEAD OF DEPARTMENT OF Civil Engineering.

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Strength of Corrugated Iron.

Owing to the scarcity of timber and the decrease in the cost of steel, the so-called corrugated iron has come into general use for the covering of roofs and the sides of buildings. This material is often, and perhaps ordinarily, called corrugated iron, although it is now always made of steel.

The necessity of having manufacturing and other buildings fire proof, has also helped to bring corrugated steel into use. It is light and if kept well painted is durable, and it is easily fastened to the building. Owing to these advantages it is generally used on mill buildings.

Corrugated steel is made by running a steel sheet through rolls, giving it a cross section of the general form shown in Fig. 1. Different

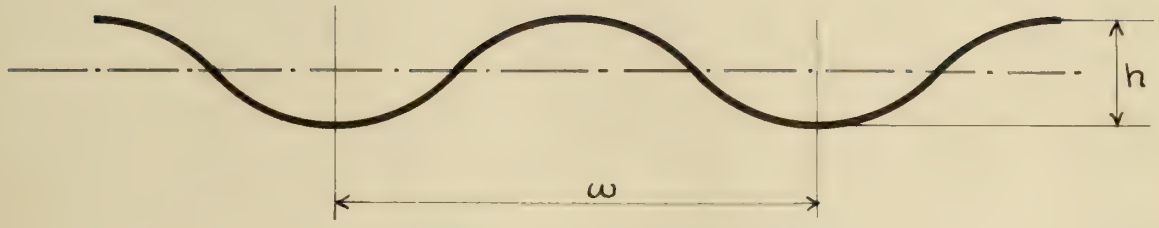


Fig. 1

widths of corrugations measured from crest to crest of wave, or w as shown in Fig. 1, varying from $\frac{3}{16}$ to 5 inches, can be obtained, and different thickness varying from No. 16 to No. 28, U. S. Standard Gage, or from 0.065 to 0.014 inches. The standard width of corrugations is $2\frac{1}{2}$ inches, and can be obtained in sheets $26\frac{1}{2}$ inches wide and in lengths varying from 5 to 12 feet. This is the size generally used for roofing and siding. Corrugating it gives the sheet considerable transverse strength when the supports are perpendicular to the corrugations. After it is corrugated the sheets are painted, then bound into bundles ready for sale or shipment.

Corrugated steel has to support

transverse loads of snow, sleet and wind on the roof and of wind on the sides of buildings, and hence it is important to know the laws of its transverse strength, so that it may be used safely but not extravagantly.

The fundamental formula for flexure is $M = \frac{SI}{c}$, where M = resisting moment, S = modulus of rupture, I = moment of inertia, and c = the distance to the extreme fibre. The difficulty in applying this formula to corrugated iron is in deducing a value for I. There are three methods applicable in the computation of I: one by considering the curves of the corrugations as cycloids, the second by considering them as parts of parabolas, and the third by assuming the corrugations to be arcs of circles. These three methods for finding I will be developed and the value for M worked out for each.

Methods of Calculating I

1. Cycloid Method.

Rankine seems to have been the

first one to investigate this subject, at least his formula is the only one employed in this country in computing the strength of corrugated steel, and is to be found in all structural steel hand-books. His investigations were made in 1864. He assumed the curve of the cross section of the corrugation to be arcs of cycloids as this curve has geometrical properties which causes the computation of its moment of inertia to be comparatively simple. EAF in Fig. 2 is assumed to

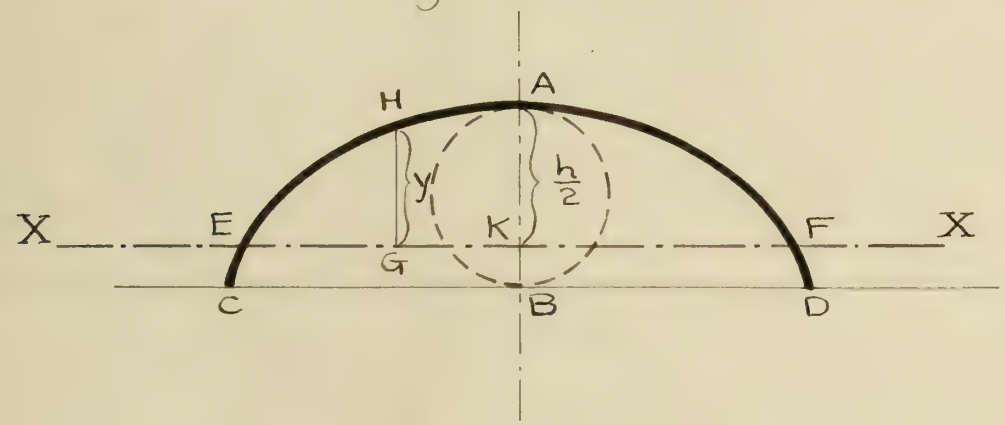


Fig 2

represent the part of the ridge of the corrugated steel above the neutral axis XX, and CAD represents a complete cycloid, traced by a point in the circle AB, as it rolls on the base line CD. The neutral axis XX is drawn

parallel to the base line CD and up such a distance as to cut off the arc EAF, so that the depth AK of this arc bears to the breadth EF the same proportion as the depth of the corrugation bears to its width.

Let w = width of one complete corrugation measured from ridge to ridge, and let h = depth from ridge to furrow, see Fig. 1; then $EF = \frac{w}{2}$, and $AK = \frac{h}{2}$. From the property of the cycloid we have the distance HG, of any point H in the arc, from a chord EF, varies proportionally to the product HE x HF of the two parts into which the arc EAF is divided by the point H. This may be expressed by the following equation. Let arc AE = AF = c, and the arc AH = s, and HG = y, then we have $y : \frac{h}{2} :: (c-s)(c+s) : c^2$ and therefore $y = \frac{h}{2}\left[1 - \frac{s^2}{c^2}\right]$.

Taking ds as an elementary arc near the point H we have the moment of inertia of this arc ds about the axis XX, equal to $y^2 ds$: and for the whole arc EA,

$$I' = \int_0^c y^2 ds = \int_0^c \frac{h^2}{4}\left[1 - \frac{s^2}{c^2}\right]^2 ds,$$

Squaring the quantity in the parenthesis and integrating we have,

$$I' = \frac{h^2}{4}\left[s - \frac{2s^3}{3c^2} + \frac{s^5}{5c^4}\right]_0^c = \frac{h}{4}\left(\frac{8}{15}c\right)$$

$$\therefore \quad I' = \frac{2}{15}h^2c.$$

Dividing this equation through by c we get,

$$\frac{I'}{c} = \frac{2}{15}h^2$$

but $\frac{I'}{c}$ = the square of the radius of gyration = r^2 or,

$$\frac{I'}{c} = r^2 = \frac{2}{15}h^2.$$

Now to find the moment of inertia I of the cross section of a sheet of corrugated iron, we must multiply the area of the cross section by r^2. Let A be that area then

$$I = \frac{2}{15}h^2A.$$

Rankine used the following approximation to calculate the sectional area of the corrugated sheet.

$$A = bt,$$

where b = the breadth of the sheet and t=

the thickness of the metal.
Substituting this value of A in the above equation for I gives

$$I = \frac{2}{15} h^2 b t,$$

and substituting this value for I in the previous equation for M, gives

$$M = \frac{SI}{c} = \frac{S \times \frac{2}{15} h^2 b t}{c},$$

but $c = \frac{h}{2}$

$$\therefore \underline{\underline{M = \frac{4}{15} S h b t}}$$

It is clear that taking $A = b_1 t$ is an approximation, for the actual area of the cross-section is $A = b_1 t'$, where t' is the virtual thickness, that is the thickness of a cross section having the same breadth b_1 and an area equal to the corrugated one; i.e. area a, b, c, d, e, = area a', b', c', d', see Fig. 3. The approximation

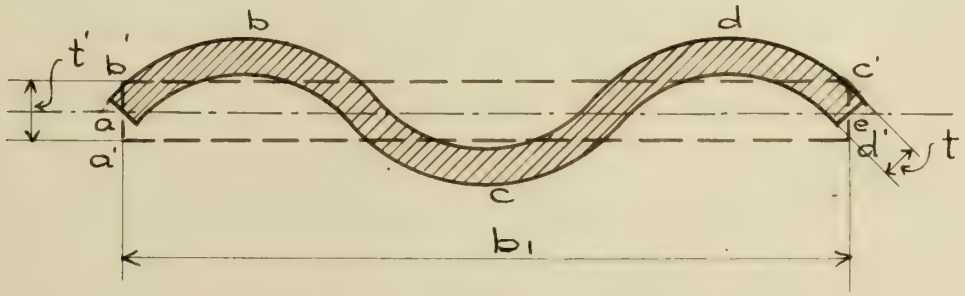


Fig 3.

used always gives too small an area, and therefore gives low values for the strength of the sheets and an extravagant use of the metal in designing.

The actual moment of inertia may be found by multiplying the value of I as deduced above by the ratio $\frac{t'}{t}$ giving the equation below as a more accurate one.

$$I = \frac{2}{5} h^2 b t \left(\frac{t'}{t}\right) = \frac{2}{15} h^2 b t'.$$

Hence the exact formula for M is,

$$M = \frac{4}{15} S h b t'.$$

But since it is a difficult matter to obtain the virtual thickness, it would be better if we had a more exact formula in terms of the actual instead of the virtual thickness. By making a very slight approximation the writer has been able to deduce such a formula.

From our definition of virtual thickness,

$$\frac{t'}{t} = \frac{\text{Length of Arc}}{\text{Length of Chord}} = \frac{CAD}{CD} \quad (\text{see Fig 2});$$

but from the cycloid, CAD = 8r, where

r = the radius of the generating circle AB, and $CD = 2\pi r$.

$$\therefore \frac{t'}{t} = \frac{8r}{2\pi r} = \frac{4}{\pi}, \text{ and } t' = 1.273t$$

Substituting this value of t' in the equation for M we have,

$$M = \frac{4}{15} S h b t = \frac{4}{15} S h b (1.273 t) = 0.340 S h b t$$

$$\underline{\underline{M = 0.340 S h b t}}$$

For a concentrated load at the center,

$$M = \frac{WL}{4} = \frac{4}{15} S h b t' = 0.340 S h b t$$

$$\therefore \underline{W = \frac{16}{15} \frac{S h b t}{l}}, \text{ or } \underline{W = \frac{1.36 S h b t}{l}}$$

For a uniformly distributed load,

$$M = \frac{Wl}{8} = \frac{4}{15} S h b t' = 0.340 S h b t$$

$$\therefore \underline{W = \frac{32}{15} \frac{S h b t'}{l}} \text{ or } \underline{W = \frac{2.72 S h b t}{l}}$$

2. Parabola Method.

This time we will consider the

curves of the corrugation as arcs of a parabola and find the moment of inertia I. This method does not give us such a simple equation for I as the one before, but it involves less approximation.

CAD, Fig. 4 represents part of a

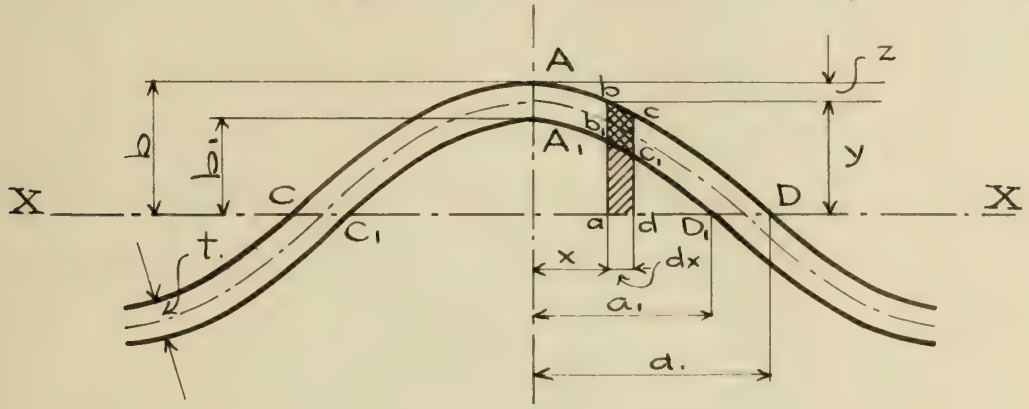


Fig. 4.

section of the corrugated steel of thickness t. We first find the moment of inertia of the entire area CAD about the axis XX and from this we substract the moment of the area $C_1 A_1 D_1$ about the same axis, the result being the moment of inertia of the of the required. The moment of inertia of the elementary area abcd about the axis XX may be expressed by the equation,

$$dI_a = \frac{y^3 dx}{3},$$

and summing this up over the entire area we get,

$$I_a = \frac{1}{3}\int_{-a}^{+a} y^3 dx .$$

From the geometrical properties of the parabola we know that the tangential offset to a point on the curve varies as the square of the perpendicular erected from the diameter to the point, or expressed by a mathematical formula:

$$z : b :: x^2 : a^2 \qquad \text{but} \quad z = b - y$$
$$b - y : b :: x^2 : a^2$$
$$a^2(b-y) = bx^2,$$

solving for y we get

$$y = \frac{a^2 b - bx^2}{a^2},$$

and $$y^3 = \frac{a^6 b^3 - 3a^4 b^3 x^2 + 3a^2 b^3 x^4 - b^3 x^6}{a^6},$$

Substituting this value of y^3 in the equation for I_a above we get,

$$I_a = \frac{1}{3a^6}\int_{-a}^{+a} \left[a^6 b^3 x - 3a^4 b^3 x^2 + 3a^2 b^3 x^4 - b^3 x^6\right] dx$$

and by integrating we get,

$$I_a = \frac{1}{3a^6}\left[a^6b^3x - \frac{3a^4b^3x^3}{3} + \frac{3a^2b^3x^5}{5} - \frac{b^3x^6}{7}\right]_{-a}^{+a}$$

$$= \frac{2}{3a^6}\left[a^7b^3 - a^7b^3 + \frac{3}{5}a^7b^3 - \frac{1}{7}a^7b^3\right]$$

$$I_a = \frac{32}{105}\,ab^3$$

Likewise for the area $C_1 A_1 D_1$,

$$I_{a_1} = \frac{32}{105}\,a_1b_1^3.$$

The moment of inertia for the section $CADD_1A_1C_1$ is $I' = I_a - I_{a_1}$ or,

$$I' = \frac{32}{105}\left(ab^3 - a_1b_1^3\right).$$

I for a complete corrugation from crest to crest of a wave is twice the above amount or,

$$I = \frac{64}{105}\left(ab^3 - a_1b_1^3\right).$$

Now that we have I we can find the value of the resisting moment as before.

$$M' = \frac{SI}{c}, \text{ but } c = b \text{ and,}$$

$$\therefore M' = \frac{64}{105}\,\frac{S(ab^3 - a_1b_1^3)}{b}.$$

For a sheet of n corrugations

$$M = \frac{64 n S (a b^3 - a_1 b_1^3)}{105 b}.$$

For a concentrated load at the center,

$$M = \frac{Wl}{4} = \frac{64 n S (a b^3 - a_1 b_1^3)}{105 b},$$

$$\therefore \quad W = \frac{256 n S (a b^3 - a_1 b_1^3)}{105 b l}$$

For a uniformly distributed load,

$$M = \frac{Wl}{8} = \frac{64 n S (a b^3 - a_1 b_1^3)}{105 b}$$

$$\therefore \quad W = \frac{512 n S (a b^3 - a_1 b_1^3)}{105 b l}$$

From Fig 4 we see that $b - b_1 = t$ and also if the metal crosses the neutral at 45°, $a - a_1 = 1.4\,t$ approximately.

3. Circular Method.

For this method the assumption is that the curves of the corrugated iron are arcs of circles. Four cases will be worked out, the metal crossing the

neutral axis at 30°, 45°, 60° and 90° respectively.

Case I - Metal Crossing Axis at 30°

Let ACE, Fig. 5, be the neutral axis,

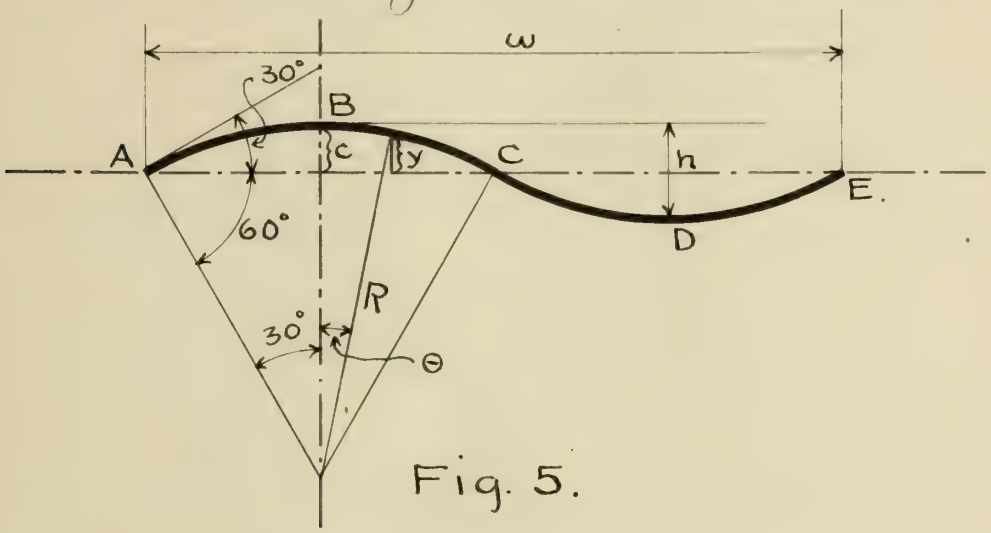


Fig. 5.

with the metal crossing it at 30°; t = the actual thickness of the metal, R = the radius of the circular arc ABC, h = the depth of corrugation from top of ridge to bottom of furrow, and w = the width of one complete corrugation from crest to crest of wave.

To find I for the section ABCDE we consider the elementary area abcd, Fig. 6. Its area is $Rtd\theta$ and its moment of inertia, dI' about the axis AE is

$Rtd\theta y^2$.

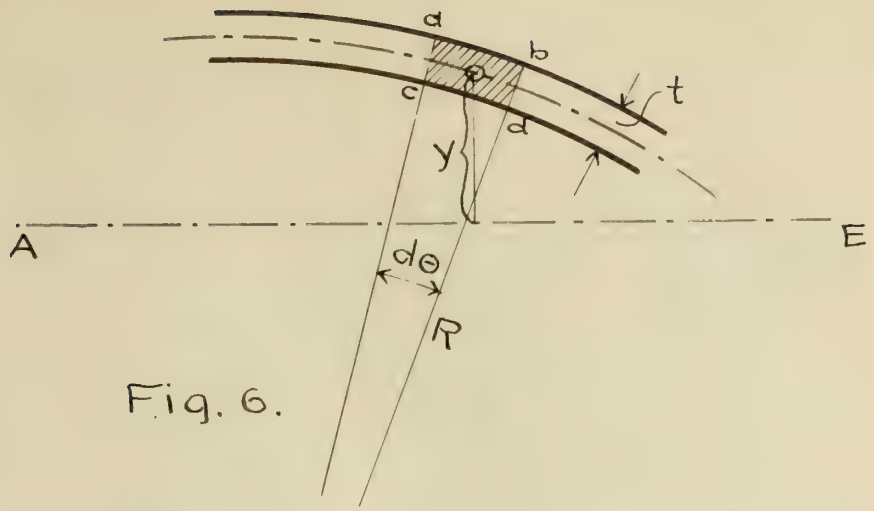


Fig. 6.

Summing this value up over the entire section we have,

$$I = 2\int_{-\frac{\pi}{6}}^{+\frac{\pi}{6}} R\,t\,y^2\,d\theta,$$

but $y = R\cos.\theta - R\cos 30° = R\left(\cos\theta - \frac{\sqrt{3}}{2}\right)$,

and $y^2 = R^2\left(\cos^2\theta - \sqrt{3}\cos\theta + \frac{3}{4}\right)$.

Substituting this value of y^2 in the equation for I above we get,

$$I = 2R^3t\int_{-\frac{\pi}{6}}^{+\frac{\pi}{6}}\left[\cos^2\theta - \sqrt{3}\cos\theta + \frac{3}{4}\right]d\theta$$

$$= 2R^3t\left[\frac{\sin\theta\cos\theta}{2} + \frac{\theta}{2} - \sqrt{3}\sin\theta + \frac{3\theta}{4}\right]_{-\frac{\pi}{6}}^{+\frac{\pi}{6}}$$

$$= 4R^3t\left[\frac{5\pi}{24} - \frac{3\sqrt{3}}{8}\right] = 0.02\,R^3t.$$

$\therefore I = 0.02 R^3 t.$

In order to compare this formula with Rankine's, and also to establish the fact that when the formula for M is given in terms of virtual thickness it is independant of the angle that the metal makes with the axis, we will get the value of I in terms of t'. If t' is the vertual thickness then we have from our defination,

$$\frac{t'}{t} = \frac{\text{Length of Arc}}{\text{Length of chord}} = \frac{\frac{\pi}{3} R}{R} = \frac{\pi}{3}$$

and $t = \frac{3t'}{\pi} = 0.955\, t'.$

Substituting this value of t in the equation for I above we have,

$$I' = 0.02 \times .955\, R^3 t' = 0.019\, R^3 t'$$

$$I' = 0.019\, R^3 t'$$

We now have the moment of inertia for one complete corrugation from ridge to ridge in terms of the actual thickness and the radius of curvature and also in terms of the virtual thickness and the

radius of curvature.

To find the resisting moment for one complete corrugation, or for the section ABCDE, Fig 5,

$$M = \frac{SI}{c} \quad \text{but} \quad c = \frac{h}{2}$$

and $$\frac{h}{2} = R - R\cos 30° = R\left(1 - \frac{\sqrt{3}}{2}\right) = 0.134\,R.$$

or $$R = 3.731\,h.$$

$$\therefore M_o = \frac{S \times 0.02\,R^3 t}{0.134\,R} = 0.149\,SR^3 t.$$

If n = the total number of corrugations in the sheet, and b represents the width of the sheet,

$$n = \frac{b}{w}, \quad \text{but } w = 4R\sin 30° = 2R.$$

and $$n = \frac{b}{2R}.$$

Then the total resisting moment M in terms of n, and the depth of corrugation h is obtained as follows: Substitute in the equation for M_o above the value of R_o in terms of h and multiply by n, to get the resisting moment for n corrugations

and we have, $M = M_o \times n$, or,

$$M = 0.149\, n\, S\, (3.731\, h)^2 t = 2.076\, n h^2 S t.$$

$$\underline{M = 2.076\, n h^2 S t.}$$

If we wish this in terms of the breadth of sheet, we substitute the value of n in terms of b, or $n = \frac{b}{2R}$, and then,

$$M = \frac{2.076\, b h^2 S t}{2R} = \frac{2.076 b h^2 S t}{2 \times 3.731\, h}$$

$$\therefore \underline{M = 0.278\, S h b t.}$$

To get this last formula in terms of the virtual thickness we simply substitute the value of t' in place of t or $t = 0.955\, t'$ as found above and we have,

$$M' = 0.278 \times 0.955\, S h b t' = 0.266\, S h b t'$$

$$\underline{M' = 0.266\, S h b t'.}$$

For a concentrated load at the center,

$$M = \frac{Wl}{4} = 0.278\, S h b t = 0.266\, S h b t',$$

and $W = \frac{1.112\,Shbt}{l} = \frac{1.0645\,Shbt'}{l}$

For a uniformly distributed load,

$$M = \frac{Wl}{8} = 0.2785\,Shbt = 0.2665\,Shbt',$$

and $W = \frac{2.224\,Shbt}{l} = \frac{2.128\,Shbt'}{l}$

Case II – Metal Crossing Axis at 45°.

Using the same nomenclature as in Case I and Fig. 7 below we find I

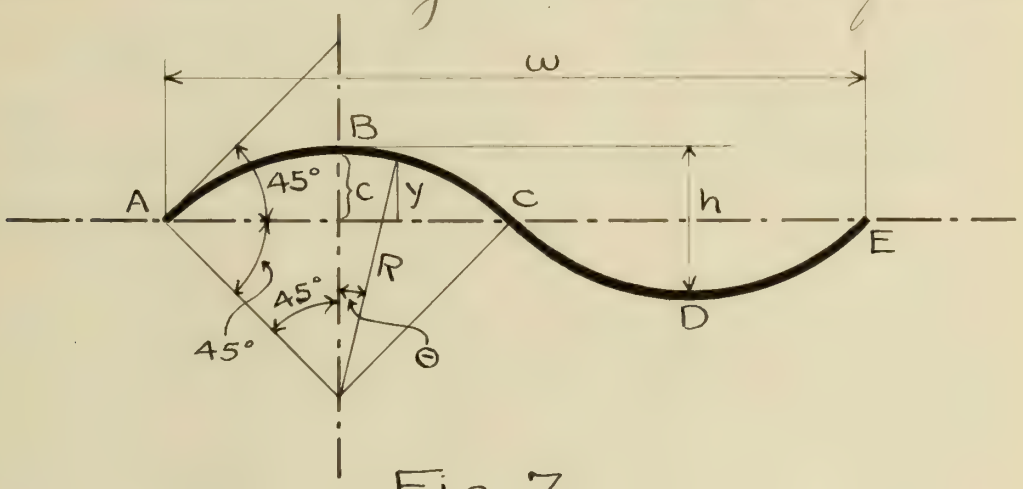


Fig. 7.

for the section ABCDE, Fig. 7, in a similar manner as for Case I.

$$I = 2\int_{-\frac{\pi}{4}}^{+\frac{\pi}{4}} Rty^2 d\theta,$$

but $y = R\cos\theta - R\cos 45° = R(\cos\theta - \frac{1}{\sqrt{2}})$,

and $y^2 = R^2[\cos^2\theta - \sqrt{2}\cos\theta + \frac{1}{2}]$.

Substituting this value of y^2 in the equation for I as before we have,

$$I = 2R^3t\int_{-\frac{\pi}{4}}^{+\frac{\pi}{4}}[\cos^2\theta - \sqrt{2}\cos\theta + \tfrac{1}{2}]\,d\theta$$

$$= 2R^3t\left[\frac{\sin\theta\cos\theta}{2} + \frac{\theta}{2} - \sqrt{2}\sin\theta + \frac{\theta}{2}\right]_{-\frac{\pi}{4}}^{+\frac{\pi}{4}}$$

$$= 4R^3t\left[\frac{\pi}{4} - \frac{3}{4}\right] = 0.1416\,R^3t$$

$\therefore \underline{I = 0.1416\,R^3t.}$

To obtain I in terms of the virtual thickness we have,

$$\frac{t'}{t} = \frac{\text{Length of Arc}}{\text{Length of Chord}} = \frac{\frac{\pi}{2}R}{\sqrt{2}R} = \frac{\pi}{2\sqrt{2}}$$

and $t = \frac{2\sqrt{2}\,t'}{\pi}$

If I' is the moment of inertia for one complete corrugation ABCDE Fig. 7, in terms of t' then,

$$I' = 0.1416 \times \frac{2\sqrt{2}}{\pi} R^3t' = 0.127\,R^3t'$$

$\therefore I' = 0.127 R^3 t'$

The resisting moment then is $M_o = \frac{SI}{c}$,

but $c = \frac{h}{2}$ and $\frac{h}{2} = R - R\cos 45° = R\left(1 - \frac{1}{\sqrt{2}}\right)$

$\frac{h}{2} = .293 R$ and $h = .586 R$ or $R = 1.706 h$.

$\therefore M_o = \frac{S \times .1416 R^3 t}{.293 R} = 0.484 S R^2 t.$

If n = the total number of complete corrugations in the sheet, and b = the width of the sheet then,

$n = \frac{b}{w}$ but $w = 4R \sin 45° = 2\sqrt{2} R$

and $n = \frac{b}{2\sqrt{2} R}$.

The resisting moment M for n corrugations is,

$M = n M_o$

or $M = 0.484 n S R^2 t.$

Substituting h for R, since $R = 1.706 h$ we have,

$M = 0.484 (1.706 h)^2 n S h^2 t = 1.413 n h^2 S t.$

$\therefore \underline{M = 1.413\, n h^2 S t.}$

To get this in terms of the breadth of sheet b, substitute, $n = \frac{b}{2\sqrt{2}} R$ or,

$$M = \frac{1.413\, b h^2 S t}{2\sqrt{2}\, R} = \frac{1.413\, b h^2 S t}{2\sqrt{2} \times 1.706 h}$$

and $\underline{\underline{M = 0.293\, S h b t.}}$

Substituting the value of t in terms of t' we get the value of M' in terms of the vertical thickness.

$$M' = 0.293\, S h b \left(\frac{2\sqrt{2}}{\pi}\right) t' = 0.263\, S h b t'$$

$$\underline{M' = 0.263\, S h b t'.}$$

For a concentrated load at the center,

$$M = \frac{Wl}{4} = 0.293\, S h b t = 0.263\, S h b t'$$

and $$W = \frac{1.172\, S h b t}{l} = \frac{1.052\, S h b t'}{l}.$$

For a uniformly distributed load.

$$M = \frac{Wl}{8} = 0.293\, S h b t = 0.263\, S h b t'$$

and $$W = \frac{2.344\, S h b t}{l} = \frac{2.104\, S h b t'}{l.}$$

Case III - Metal Crossing Axis at 60°

Using the same nomenclature as in Case I and Fig. 8. below we find

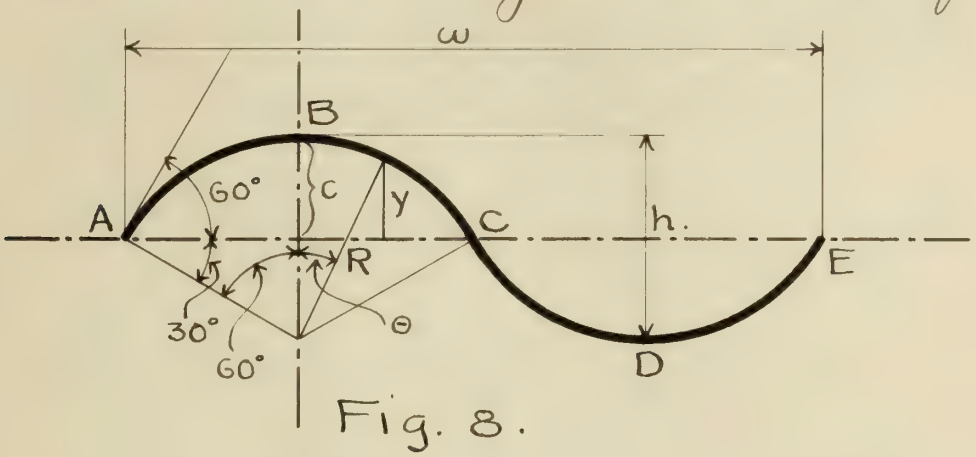


Fig. 8.

I for the section ABCDE, Fig. 8, in a similar manner as for Case I.

$$I = 2\int_{-\frac{\pi}{3}}^{+\frac{\pi}{3}} \left[R + y^2\right] d\theta$$

but $y = R\cos\theta - R\cos 60° = R\left(\cos\theta - \frac{1}{2}\right)$

and $y^2 = R^2\left[\cos^2\theta - \cos\theta + \frac{1}{4}\right]$.

Substituting this value of y^2 in the equation for I above we have,

$$I = \int_{-\frac{\pi}{3}}^{+\frac{\pi}{3}} \left[2R^3 + \left(\cos^2\theta - \cos\theta + \frac{1}{4}\right)\right] d\theta$$

$$= 2R^3 + \left[\frac{\sin\theta\cos\theta + \theta}{2} - \sin\theta + \frac{\theta}{4}\right]_{-\frac{\pi}{3}}^{+\frac{\pi}{3}}$$

$$= 4R^3t\left[\frac{\sqrt{3}}{8} + \frac{\pi}{4} - \frac{\sqrt{3}}{2}\right] = 0.544\,R^3t$$

$$\therefore \underline{I = 0.544\,R^3t.}$$

To get I' in terms of the virtual thickness we do as before,

$$\frac{t'}{t} = \frac{\text{Length of Arc}}{\text{Length of Chord}} = \frac{\frac{2\pi}{3}R}{\sqrt{3}R} = 1.209$$

$$t = 0.827\,t'.$$

$$I' = 0.544R^3 \times 0.827t = 0.450\,R^3t'$$

$$\therefore I' = 0.450\,R^3t'.$$

The resisting moment is then obtained by substituting the value deduced for I in the formula $M = S\frac{I}{c}$.

$$c = \frac{h}{2} \text{ and } \frac{h}{2} = R - R\cos 60° = \frac{R}{2}$$

or $$R = h.$$

$$\therefore M_o = \frac{S \times 0.554\,R^3t}{\frac{h}{2}} = \frac{S \times 0.554\,h^3t}{\frac{h}{2}}$$

$$M_o = 1.087\,Sh^2t.$$

For n corrugations, $M = nM_o$

or $\underline{M = 1.087\, n\, S h^2 t}$

but $n = \frac{b}{w}$ and $w = 4R \sin 60° = 3.464\, R$

or $n = \frac{b}{3.464\, h}$

$$\therefore M = \frac{1.087\, S h^2 b t}{3.464\, h} = 0.312\, S h b t$$

$$\underline{\underline{M = 0.312\, S h b t}}.$$

Substituting the value of t in terms of t' we get the value of M' in terms of the virtual thickness.

$$M' = 0.312\, S h b \times .827\, t' = 0.259\, S h b t'$$

$$\underline{M' = 0.259\, S h b t'.}$$

For a concentrated load at the center,

$$M = \frac{Wl}{4} = 0.312\, S h b t = 0.259\, S h b t'$$

or $$W = \frac{1.248\, S h b t}{l} = \frac{1.036\, S h b t'}{l}.$$

For a uniformly distributed load.

$$M = \frac{Wl}{8} = 0.312\,Shbt = 0.259\,Shbt'$$

or $$W = \frac{2.496\,Shbt}{l} = \frac{2.072\,Shbt'}{l}.$$

Case IV - Metal Crossing Axis at 90°.

When the metal crosses the neutral axis at 90°, the section ABC, Fig. 9, is a

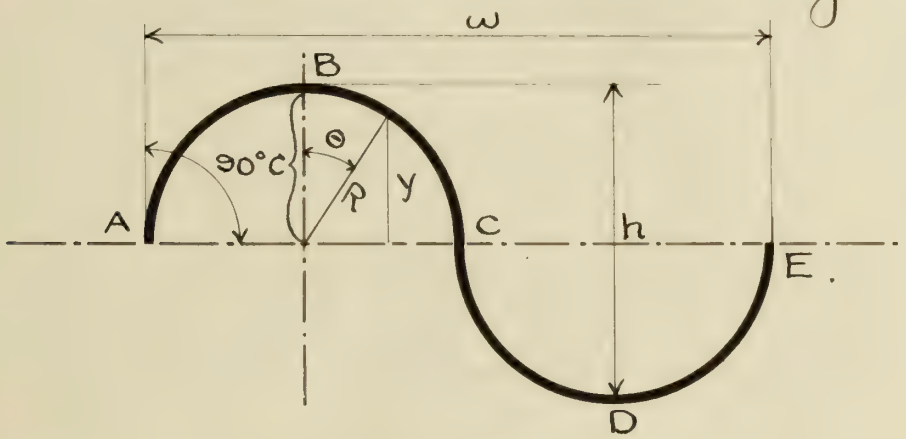


Fig. 9.

semicircle and the computations are much simpler, though similar to the previous cases.

$$I = 2\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} Rty^2 d\theta$$

but $y = R\cos\theta$ and $y^2 = R^2\cos^2\theta$,

and $$I = 2R^3t\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 2R^3t\left[\frac{\sin\theta\cos\theta}{2} + \frac{\theta}{2}\right]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}$$

$$= \pi R^3 t$$

$$\therefore \underline{I = 3.142\ R^3t}.$$

The relation between the virtual and actual thickness is,

$$\frac{t'}{t} = \frac{\text{Length of Arc}}{\text{Length of Chord}} = \frac{\pi R}{2R} = \frac{\pi}{2},$$

and $t = \frac{2t'}{\pi} = 0.636\,t'$.

If I' is the moment of inertia of the section ABCDE in terms of the virtual thickness then,

$$I' = \frac{2\pi R^3 t'}{\pi} = 2R^3 t'.$$

$$\therefore \underline{I = 2R^3 t'}.$$

Substituting these values of I in the formula for the resisting moment $M_o = \frac{SI}{c}$ we have,

$$M_o = \frac{S \times \pi R^3 t}{c} \quad \text{but } c = R = \frac{h}{2}$$

$$\therefore M_o = \frac{\pi S\left(\frac{h}{2}\right)^3 t}{\frac{h}{2}} = \frac{\pi}{4} S h^2 t.$$

The moment of inertia M for n corrugations is $M = nM_o$ and

$$M = n\frac{\pi}{4} S h^2 t = 0.785 n h^2 S t.$$

$$\underline{M = 0.785 n h^2 S t},$$

but $n = \frac{b}{w}$ and $w = 2h$ then,

$$n = \frac{b}{2h}.$$

Substituting this value for n in the formula above we get

$$M = \frac{\pi b S h^2 t}{4 \times 2h} = 0.393 S h b t.$$

$$\underline{\underline{M = 0.393 S h b t.}}$$

Introducing the virtual thickness in place of the actual thickness we have

$$M' = \frac{\pi S h b \times 2t'}{8\pi} = 0.25 S h b t'.$$

$$M' = 0.250 S h b t'.$$

For a concentrated load at the center,

$$M = \frac{Wl}{4} = 0.393\ Shbt = 0.250\ Shbt'$$

or $$W = \frac{1.572\ Shbt}{l} = \frac{1.00\ Shbt'}{l}.$$

For a uniformly distributed load,

$$M = \frac{Wl}{8} = 0.393\ Shbt = 0.250\ Shbt'$$

or $$W = \frac{3.142\ Shbt}{l} = \frac{2.00\ Shbt'}{l}$$

TABLE I.
Summary of Formulas.

	1.	2.	3.	4.	5.
Curve Assumed.	I. Interms of the Actual Thickness	I'. Interms of the Virtual Thickness	M. Interms of the Actual Thickness	M'. Interms of the Virtual Thickness.	M. Interms of the number of Corrugations
Circle Metal Crossing Axis at 30°	$1.035\,nh^3t.$	$0.988\,nh^3t'$	$0.278\,Shbt.$	$0.266\,Shbt'$	$2.076\,nh^2St$
Circle Metal Crossing Axis at 45°	$0.703\,nh^3t$	$0.631\,nh^3t'$	$0.293\,Shbt.$	$0.263\,Shbt'$	$1.413\,nh^3St$
Circle Metal Crossing Axis at 60°	$0.544\,nh^3t.$	$0.450\,nh^3t'$	$0.312\,Shbt.$	$0.259\,Shbt'$	$1.087\,nh^2St$
Circle Metal Crossing Axis at 90°	$0.393\,nh^3t.$	$0.250\,nh^3t'$	$0.393\,Shbt.$	$0.255\,Shbt'$	$0.785\,nh^2St.$
Clycloid	$0.170\,h^2bt.$	$0.133\,h^2bt.$	$0.340\,Shbt.$	$0.267\,Shbt'$	
Parabola	$\frac{64}{105}n(ab^3-a_1b_1^3)$		$\frac{64nS}{105b}(ab^3-a_1b_1^3)$		

TABLE II

Formula for Uniform and Concentrated Breaking Loads.

No.	Curve Assumed	Breaking Uniform Load W	Breaking Concentrated Load W.
1	Circle. Metal Crossing Axis at 30°	$\frac{2.224\ Shbt}{l}$	$\frac{1.112\ Shbt}{l}$
2	Circle Metal Crossing Axis at 45°	$\frac{2.344\ Shbt}{l}$	$\frac{1.172\ Shbt}{l}$
3	Circle Metal Crossing Axis at 60°	$\frac{2.496\ Shbt}{l}$	$\frac{1.248\ Shbt}{l}$
4	Circle Metal Crossing Axis at 90°	$\frac{3.142\ Shbt}{l}$	$\frac{1.572\ Shbt}{l}$
5	Clycloid	$\frac{2.720\ Shbl}{l}$	$\frac{1.36\ Shbt}{l}$
6	Parabola.	$\frac{4.88\ nS(ab^3-a_1b_1^3)}{bl}$	$\frac{2.44\ nS(ab^3-a_1b_1^3)}{bl}$

A glance down column 4 of Table I shows that if the virtual thickness is used the formula is general for any depth of corrugation as long as the curves are approximately circular arcs.

Rankine's formula $M = \frac{4}{15} Shbt = 0.267 Shbt$, developed under the assumption that the curves of the corrugations were arcs of cycloids, is seen by inspection of Table I to correspond to the case where the metal crosses the axis at about 20°. Apparently this angle is never as low as 20°, at least not in corrugated steel made in this country. The writer found that for two different manufacture and a number of different sizes the mean angle that the metal makes with the axis is 38°; so it is evident that Rankine's formula gives results too great, even extravagant.

Experimental Investigation.

To check up the preceding mathematical investigation the writer submitted some sheets of corrugated steel to transverse stress in a testing machine. The sheets

were 12 inches wide and 48 inches long, and were supported at the ends upon steel beams, with bearings similar to those they will have in actual use except that the ends were not fastened down as they are on the building. The load was applied through a knife edge which rested upon a strip of wood 1 inch wide, whose underside conformed to the shape of the corrugations. To insure a uniform distribution of the pressure a thin layer of fine sand was put under the wood strip.

The deflections were measured for increments varying from 10 up to 100 pounds, up to the elastic limit. The load was then gradually increased until the sheets buckled.

The ultimate tensile strength of the metal was determined by testing a strip cut from several sheets of each set. The results of these tests are given in Table III. There were three different sets of corrugated steel, and each set seemed to be made of metal having a different tensile strength.

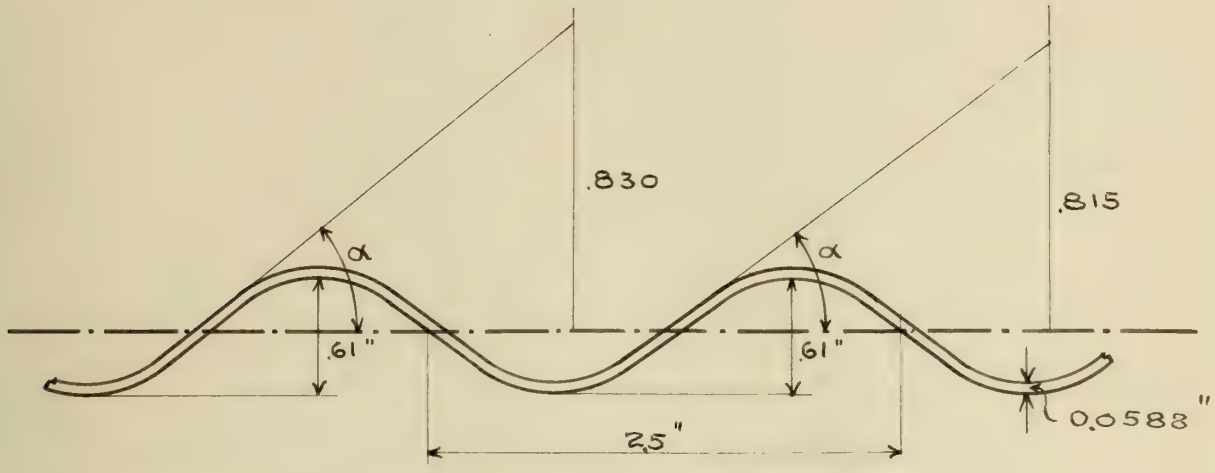
The details of a few of the tests will be given, and a summary of all the tests is given in Table IV.

TABLE III.

Ultimate Tensile Strength of the Steel.

Set	Sheet No.	Cross Section of Test Speciman Inches.	Area Sq. in.	Breaking Load lbs.	Tensile Strength. lbs. per sq. in.
1	1	0.026 x 1.39	0.0362	1,850	51,000
	2	0.036 x 1.26	0.046	2,300	50,500
2	3	0.038 x 1.34	0.051	3,400	66,750
	4	0.038 x 0.98	0.0372	2,500	67,250
3	5	0.067 x 1.095	0.0739	4,300	58,300
	6	0.061 x 1.30	0.0793	4,550	57,400
	7	0.067 x 1.09	0.073	4,250	58,200

1st Test.



Actual cross section of Sheet I showing method of finding depth of corrugations and angle, α, that metal makes with axis.
Tan α = .830 = .815 etc Mean α = 39° 11'

Width of sheet = 12 inches.
Width of corrugations = 2½ inches.
Mean depth of corrugations h = 0.6025 inches.
Mean thickness of metal t = 0.0588 inches.
Distance between supports l = 44 inches.
Ultimate tensile strength, see Table III, Set 3, S = 58,000 lbs. per sq. in.
Mean angle of metal with axis = 39° 11"

Load	Deflection
50 lbs.	0.151 in.
100 "	0.262 "
150 "	0.349 "

200 lbs	0.445 in.
300 "	0.613 "
400 "	0.785 "
500 "	0.900 "
600 "	1.188 "
630 "	5.000 " Buckled.

Interpolating between formulas 1 and 2, Table II, the value for the concentrated center breaking load is,

$$W = \frac{1.149\, S h b t}{l},$$

and substituting the above values in this formula we get,

$$W = \frac{1.149 \times 58000 \times 0.6025 \times 1\tfrac{1}{2} \times 0.0588}{44} = 643 \text{ lbs.}$$

In other words the computed breaking center load is 643 lbs., while the observed value was 630 lbs. - a very satisfactory agreement.

2nd Test.

Width of sheet = b = 12 inches.
Width of corrugations = $2\frac{1}{2}$ inches.

Mean depth of corrugations h = 0.612 inches.
Mean thickness of metal t = 0.0568 inches.
Distance between supports l = 44. inches.
Ultimate tensile strength S = 58000 lbs. per sq. in.
Mean angle of metal with axis = 39° 10'

Load	Deflection.
50 lbs	0.115 in
100 "	0.225 "
150 "	0.314 "
200 "	0.406 "
250 "	0.505 "
300 "	0.596 "
400 "	0.772 "
500 "	0.992 "
525 "	1.059 "
550 "	1.172 "
575 "	1.40 "
600 "	1.80 "
630 "	5.00 " Buckled.

Interpolating as before we find,

$$W = \frac{1.149\, S h b t}{l}, \text{ or}$$

$$W = \frac{1.149 \times 58000 \times 0.612 \times 12 \times 0.0568}{44} = 632 \text{ lbs.},$$

which agrees almost exactly with the actual load.

3rd Test.

Width of sheet $b = 12$ inches
Width of corrugations = $2\frac{1}{2}$ inches
Mean depth of corrugations $h = 0.625$ inches
Mean thickness of metal $t = 0.066$ inches
Distance between supports $l = 44.$ inches
Ultimate tensile strength $S = 58000$ lbs. per sq. in.
Mean angle of metal with axis = 39° 30'

Laad		Deflections.	
50	lbs	0.130	in.
100	"	0.221	"
150	"	0.306	"
200	"	0.395	"
250	"	0.481	"
300	"	0.572	"
400	"	0.788	"
450	"	0.908	"
475	"	0.985	"
500	"	1.094	"
525	"	1.297	"
535	"	1.355	"
550	"	1.60	"
650	"	2.10	"
720	"	5.00	"

$W = \frac{1.146\ Shbt}{l}$ or,

$$W = \frac{1.146 \times 58000 \times 0.625 \times 12 \times 0.066}{44} = 745 \text{ lb}$$

4th Test.

Width of sheet b = 12 inches.
Width of corrugations = 2½ inches.
Mean depth of corrugations is h = 0.606 inches.
Mean thickness of metal t = 0.0655 inches
Distance between supports l = 44. inches
Ultimate tensile strength S = 58000 lbs. per sq. in.
Mean angle of metal with axis = 40°-42'.

Load	Deflections.	
50 lbs	0.149 in.	
100 "	0.243 "	
150 "	0.321 "	
200 "	0.412 "	
255 "	0.500 "	
300 "	0.587 "	
400 "	0.790 "	
500 "	1.065 "	
550 "	1.385 "	
600 "	2.000 "	
700 "	5.000 "	Buckled.

$$W = \frac{1.152\ 5\ h\ b\ t}{l} \text{ nearly,}$$

$$W = \frac{1.152 \times 58000 \times 0.606 \times 12 \times 0.0655}{44} = 725 \text{ lbs.}$$

This is also a very satisfactory agreement.

TABLE IV.
Summary of Experiments.

No	Width of Corrugat. Inches	Depth h. Inches	Thickness t Inches	Angle α	Span l Inches	Ultimate Tensile strength of Metal S	Breaking Load.		
							Actual lbs.	Theoretical	
								Author's $W=\frac{CSht}{l}$	Rankine's $W=\frac{16Sht}{15l}$
1	2.50	0.6025	0.0588	39°-11'	44.0	58,000.	630	643 lbs.	597 lbs.
2	2.50	0.612	0.0568	39°-10'	44.0	58,000.	630	632	587
3	2.50	0.625	0.0660	39°-30'	44.0	58,000.	720	745	692
4	2.50	0.606	0.0655	40°-42'	44.0	58,000.	700	725	670
5	2.88	0.650	0.0367	36°-0'	43.25	67,000.	500	505	475
6	2.88	0.650	0.0366	36°-0'	44.0	67,000.	490	494	465
7	2.50	0.630	0.0366	36°-0'	44.0	50,000.	350	358	335
8	2.50	0.610	0.0365	36°-0'	44.0	50,000.	340	344	324
9	1.25	0.270	0.0365	36°-0'	24.0	50,000.	300	281	262
10	1.25	0.270	0.0365	36°-0'	24.0	50,000.	295	281	262
11	1.25	0.270	0.0293	36°-0'	24.0	50,000.	200	225	211
12	1.25	0.270	0.0293	36°-0'	24.0	50,000.	195	225	211
13	1.00	0.180	0.0291	36°-0'	24.0	50,000.	310	298	279
14	1.00	0.180	0.0291	36°-0'	24.0	50,000.	300	298	279
15	1.00	0.180	0.0260	36°-0'	24.0	50,000.	280	266	250
16	1.00	0.180	0.0260	36°-0'	24.0	50,000.	260	266	250

In the last four tests given in Table IV the pressure was applied at two points 12 inches apart

Conclusion.

Since it has been shown that Rankine's formula for the strength of corrugated sheets gives extravagant results and since the formula deduced by the writer agrees very closely with the results of actual tests, it is suggested that the writer's formulas should be employed instead of Rankine's.

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